Topologies and Models of DC/DC Converters

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DC/DC Converters

- **Initial criteria:**
  - voltage to voltage
  - (can be varied)
  - lossless conversion
  - being controllable
  - (voltage conversion ratio, power flow, etc.)

- The simplest converter as constrained by Kirchhoff’s laws should have
  - one inductor (current interface)
  - two switches

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Three simplest topologies

Boost converter

Buck-boost converter

Buck converter
The buck converter

- Switch $S$ is turned on and off very quickly, at a rate much greater than the output filter natural frequency. Switching period = $T$.
- Control parameter is duty cycle, $d$

\[ d = \frac{\text{duration when } S \text{ is on}}{\text{period}} = \frac{t_{\text{on}}}{T} \]

- OFF time: $i_L$ falls, and $v_L = -U$.
- ON time: $i_L$ climbs only if $E > U$, and $v_L = E - U$.
- At steady state, average $v_L$ must be 0. Thus,

\[ (E - U)D = U(1 - D) \]

- Hence, $U = D E < E$, i.e., step-down converter
The buck-boost converter

* The inductor is connected to the source for $DT$ and to the load for $(1-D)T$.
* ON time: $i_L$ climbs, and $v_L = E$.
* OFF time: $i_L$ falls, and $v_L = -U$.
* Thus, depending on the value of $D$, the output $U$ can be either larger or smaller than $E$.
* At steady state, average $v_L$ must be 0, i.e.,

$$ED = U(1 - D)$$

* Hence,

$$U = \frac{D}{1 - D}E$$

$U < E$ for all $0 < D < 0.5$

$> E$ for all $0.5 < D < 1$
The boost converter

- The inductor is connected to the source for $T$ and to the load for $(1-D)T$.
- ON time: $i_L$ climbs, and $v_L = E$.
- OFF time: $i_L$ falls, and $v_L = -(U-E)$.
- Thus, $U$ must be larger than $E$ for equilibrium to be achieved.
- At steady state, average $v_L$ must be 0, i.e.,
  \[ ED = (U - E)(1 - D) \]
- Hence,
  \[ U = \frac{E}{1 - D} > E \]
- i.e., **step-up converter**
Operating modes

- So far, we have assumed the inductor current maintains a positive value throughout the period. This operation is called **continuous conduction mode (CCM)**.

- Let’s look at the buck converter. However, if
  - the inductor is too small OR the period is too long,
  - then the inductor current could fall to zero during OFF time and the diode would be open again. This introduces an IDLING interval in which \( i_L = 0 \). This is **discontinuous conduction mode (DCM)**.
Discontinuous conduction mode

- Waveforms for the buck converter in DCM.
- Let the OFF time = \( HT \);
  - and idling time = \((1-D-H)T\)
- At steady state, the average output voltage, average inductor current, and peak inductor current can be found as

\[
H = \frac{(E - U)D}{U}
\]

\[
I_{L,\text{average}} = \frac{I_{\text{max}}(D + H)}{2} = \frac{U}{R}
\]

\[
U = \frac{2E}{1 + \sqrt{1 + \frac{8L}{RD^2T}}}
\]
Discontinuous conduction mode

For the \textit{boost converter in DCM}, we have

\[ H = \frac{ED}{U - E} = \frac{LI_{\text{max}}}{(U - E)T} \]

\[ I_{\text{in, average}} = \frac{(D + H)I_{\text{max}}}{2} \]

\[ U = \frac{E}{2} \left[ 1 + \sqrt{1 + \frac{2RD^2T}{L}} \right] \]
Mode boundary

- At the borderline, the inductor current just touches zero at the end of the period. There is no idling interval, and yet the inductor current has a momentarily zero value.

- For the buck converter, at the boundary of two modes,
  - the input power is \( P_{\text{in}} = \frac{1}{2} I_{\text{max}} DE \)
  - the output power is \( P_{\text{out}} = \frac{1}{4} I_{\text{max}}^2 R \)

- Also,
  \[
  I_{\text{max}} = \frac{DT(E - U)}{L}
  \]

- Equating the input power and output power, we get the boundary condition as

\[
L_{\text{crit}} = \frac{(1 - D)TR}{2}
\]
Conditions for CCM: $L > L_{\text{crit}}$

- Buck converter
  
  \[ L_{\text{crit}} = \frac{(1 - D)TR}{2} \]

- Buck-boost converter
  
  \[ L_{\text{crit}} = \frac{(1 - D)^2TR}{2} \]

- Boost converter
  
  \[ L_{\text{crit}} = \frac{(1 - D)^2DTR}{2} \]
Fourth order converters

- The buck converter has pulsating input current.
- The boost converter has pulsating output current.
- The buck-boost converter has both pulsating input and output currents.
- We may add filter to the source side of the buck converter, and the load side of the boost converter. etc.
- The results are some higher order converters.
4th order converters with a cutset of 2 inductors and 2 switches

- Ćuk converter
- Zeta converter
- SEPIC converter

Both input and output current are non-pulsating!
The Ćuk converter

* At steady state, it behaves like a cascade connection of a boost converter and a buck converter, with the storage capacitor as intermediate output. In CCM, we have

\[ V_c = \frac{E}{1 - D} \]

\[ U = DV_c \]

* Hence,

\[ U = \frac{D}{1 - D} E \]

* Of course, it can operate in DCM, but the inductor currents may not go to zero, though their sum is zero in the idling interval.

* It also has a special discontinuous capacitor voltage mode (DCVM).
Transformer isolated versions

- **Flyback converter (buck-boost)**
  - simple
  - storage inductor provided by transformer’s magnetizing inductance

- **Push-pull converter (buck)**
  - Better core utilization as positive and negative flux polarities are used
  - Heat dissipation shared by two switches

- **Forward converter (buck)**
  - Simple core reset
  - Limits $d < 0.5$
  - Reduced voltage stress
Transformer isolated versions

- Half-bridge converter (buck)
  - Automatic core balance
  - Voltage stress shared by two devices (low voltage ratings of devices)

- Full-bridge converter (buck)
  - Automatic core balance
  - Switches operate in pairs
  - Reduced peak current compared to half-bridge converter
General selection guideline

- **flyback & forward**
- **half bridge**
- **full bridge**

Power not easily developed at these levels.
Models

- **Average models**
  - IDEA:
    - Switching details are uninteresting
    - Focus on the low-frequency dynamics
      - Resulting models have no switch, but can be nonlinear
      - Resulting models can be linearized to produce small-signal models
Averaging

- Separate the dc/dc converter into two parts:
  - switching n-port (fast part) and
  - the remaining slow part.
- Identify the *switching n-port* and replace it by equivalent average controlled sources.
- **Essential idea:**
  - The switching n-port has high-frequency operation, and the rest can be considered very slow and hence can be regarded as “constant” while modeling the switching n-port.
Example: buck converter in CCM

- In the buck converter in CCM, the inductor current is continuous and varying slowly.
- The switching n-port (fast part) contains only $D$ and $S$.
- So, when we model this switching n-port, we may treat the part outside the n-port as “constant” source.
- Derive the average terminating voltage or current.
Finally, the inductor dynamics is resumed for analysis. The average model for CCM buck converter is

**Small-signal model:**

$E, I_L, U$ and $D$ are the steady state values.

$\delta x$ is the small-signal value of $x$. 

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Example: buck converter in DCM

- In the buck converter in DCM, the inductor is absorbed in the switching n-port (fast part).
- Again, when we model this switching n-port, we can treat the part outside the n-port as a "constant" source.
- Derive the average terminating voltage or current.

\[
\frac{D^2 T}{2L} (E - U) \quad \text{where} \quad M = \frac{U}{E}
\]
Example: Cuk converter in CCM

\[ E + \stackrel{L_1}{\longrightarrow} \stackrel{i_{L1}}{\longrightarrow} \stackrel{v_c}{\longrightarrow} \stackrel{L_2}{\longrightarrow} \stackrel{i_{L2}}{\longrightarrow} \stackrel{C}{\longrightarrow} \stackrel{R}{\longrightarrow} \stackrel{U}{\longrightarrow} \]

\[ di_{L2} - (1 - d)i_{L1} \]

\[ (1 - d)v_c \]

\[ dv_c \]
Example: Cuk converter in CCM

Average model
Small-signal model of the buck converter

- From the model, we identify two inputs: \( \delta e \) and \( \delta d \)
- The output is \( \delta u \).
- We can develop transfer functions:
  - Control-to-output t/f:
    \[
    \frac{\delta u}{\delta d} = \frac{U}{D} \left[ \frac{1 + sCr_c}{1 + s \left( (r_c + r_L||R)C + \frac{L}{r_L+R} \right) + s^2LC\frac{r_c+R}{r_L+R}} \right]
    \]
  - Input-to-output t/f:
    \[
    \frac{\delta u}{\delta e} = \frac{DR}{r_L+R} \left[ \frac{1 + sCr_c}{1 + s \left( (r_c + r_L||R)C + \frac{L}{r_L+R} \right) + s^2LC\frac{r_c+R}{r_L+R}} \right]
    \]
Small-signal dynamics of the buck converter

• BUCK CONVERTER
  • When ESRs are included, a zero at $-1/r_cC$ appears. Thus, a $+20\text{dB/dec}$ response is expected from $1/(2\pi r_cC)$ Hz.
  • A pair of complex poles at fixed locations. Thus, a $-40\text{dB/dec}$ response is expected from $1/(2\pi LC)$ Hz.
Small-signal model of the boost converter

- BOOST CONVERTER (CCM)
- Average model (nonlinear):

* Linear small-signal model:
Transfer functions for the boost converter

* Control-to-output t/f:

\[
\frac{\delta u}{\delta d} = \left[ \frac{U}{(1-D)R'} \right] \left[ \frac{(1-D)^2 R^2}{r_C + R} - r_L \right] \left[ \frac{(1 + s r_c C) \left( 1 - \frac{s L (r_L + R)}{(1-D)^2 R'^2} \right)}{1 + s \left( \frac{L}{R'} + \frac{(r_L R_c + r_c r_L + (1-D) r_c R) C}{R'} \right) + s^2 L C \left( \frac{r_c + R}{R'} \right)} \right]
\]

where \( R' = \frac{(1-D)^2 R^2}{R + r_c} + (1-D)(r_c \| R) + r_L \)

* Input-to-output t/f:

\[
\frac{\delta u}{\delta e} = \left[ \frac{(1-D) R}{R'} \right] \left[ \frac{(1 + s r_c C)}{1 + s \left( \frac{L}{R'} + \frac{(r_L R + r_c r_L + (1-D) r_c R) C}{R'} \right) + s^2 L C \left( \frac{r_c + R}{R'} \right)} \right]
\]
Small-signal response of the boost converter

* A right-half-plane zero exists in the control-to-output transfer function of the boost converter!

\[ \omega_{RHP} = \frac{1}{L} \left[ \frac{(1 - D)^2 R^2}{R + r_c} - r_L \right] \approx \frac{(1 - D)^2 R}{L} \]

* The complex poles are not fixed, but depend on the duty cycle \( D \).

\[ \omega_s = \sqrt{\frac{R'}{(r_c + R)LC}} \approx \frac{1 - D}{\sqrt{LC}} \]

* The ESR zero still exists at \( 1/2\pi r_c C \) Hz.
In the time domain, a non-minimum phase response is characterized by an initial momentarily drop in output when a step increase in duty cycle is applied. The output eventually rises.

The physical origin can be easily understood from the structure of the boost converter.

Increasing the duty cycle means that the inductor is charged for a longer interval. This makes the output capacitor discharge for a longer time. But as soon as more current is supplied to the output network in subsequent cycles, the output eventually increases.

This phenomenon occurs in boost and buck-boost (flyback) converters.
**Note on DCM converters**

- Essentially first-order, because inductor current assumes zero value periodically, and its average becomes devoid of dynamics.
- The dynamics is a single-pole response, roll-off at $1/CR$. Of course ESR zero still exists.
- Easy control!

![Buck converter average model](image)
Probing further

- How does the input look?
- Can it become resistive? Under what condition?
- Any application if it has a resistive input?
  - Cheap and simple power-factor-correction converter!
Conclusion

- Simple converters: buck, buck-boost and boost converters
- Fourth order converters: Cuk converter
- Transformer isolated converters
- Average models
  - Switching details removed
  - Nonlinear models for analysis
  - Linearization to yield small-signal models
  - Transfer functions
  - Dynamic responses
  - Converters with RHP zero need special control strategies.
References


