Electronic Circuits 1

Basic Transistor Amplifiers

Contents

- Biasing
- Amplification principles
- Small-signal model development for BJT
Aim of this chapter

To show how transistors can be used to amplify a signal.
Basic idea

Step 1: Set the transistor at a certain DC level — **biasing**

Step 2: Inject a small signal to the input and get a bigger output — **coupling**
Biasing the transistor

To set the transistor to a certain DC level = To set \( V_{CE} \) and \( I_C \)

Suppose we want the following biasing condition:

\[ I_C = 10 \, \text{mA} \quad \text{and} \quad V_{CE} = 5 \, \text{V} \]

Find \( R_B \) and \( R_L \)

Start with \( V_{BE} \approx 0.7 \, \text{V} \).

Then,

\[ I_B = \frac{10 - V_{BE}}{R_B} = \frac{10 - 0.7}{R_B} \]
\[ I_C = \beta I_B = \frac{100 (10 - 0.7)}{R_B} = 10 \, \text{mA} \]

So, \( R_B = 94k\Omega \)

Also,

\[ V_{CE} = 10 - R_L I_C \]

Hence, \( 5 = 10 - 10R_L \)

So, \( R_L = 0.5k\Omega \)
This is a bad biasing circuit!

because it relies on the accuracy of $\beta$, but $\beta$ can be ±50% different from what is given in the databook.
A slightly better biasing method

Again, our objective is to find the resistors such that $I_C = 10\text{mA}$ and $V_{CE} = 5\text{V}$.

First, if $I_B$ is small, we can approximately write

$$0.6 = 10 \times \frac{R_{B2}}{R_{B1} + R_{B2}}$$

$$\Rightarrow \frac{R_{B1}}{R_{B2}} = \frac{94}{6}$$

Suppose we get $I_C = 10\text{mA}$. Then $R_L = 0.5\text{k}\Omega$.

We can start with $R_{B1} = 940\Omega$ and $R_{B2} = 60\Omega$. Such resistors will make sure $I_B$ is much smaller than the current flowing down $R_{B1}$ and $R_{B2}$, which is consistent with the assumption.

What we need in practice is to fine tune $R_{B1}$ or $R_{B2}$ such that $V_{CE}$ is exactly 5V.
A much better biasing method — emitter degeneration

Again, our objective is to find the resistors such that $I_C = 10\text{mA}$ and $V_{CE} = 5\text{V}$.

Set $V_E = 2\text{V}$, say. Then, $R_E = 2\text{V}/10\text{mA} = 0.2\text{k}\Omega$.

Surely, $R_L = 0.5\text{k}\Omega$ in order to get $V_{CE} = 5\text{V}$.

Finally, we have $V_B = V_E + 0.6$. Therefore, if $I_B$ is small compared to $I_{RB1}$ and $I_{RB2}$, we have

$$\frac{R_{B1}}{R_{B2}} = \frac{74}{26}$$

Hence, $R_{B1} = 740\Omega$ and $R_{B1} = 260\Omega$.

NOTE: $\beta$ is never used in calculation!!
Stable (good) biasing

Summary of biasing with emitter degeneration:

Choose $V_E$, $I_C$ and $V_{CE}$.

Use $V_{BE} \approx 0.6$ to get $V_B$.

Then use
\[
\frac{R_{B1}}{R_{B2}} = \frac{10 - V_B}{V_B}
\]

to choose $R_{B1}$ and $R_{B2}$ such that $I_B$ is much smaller than the current flowing in $R_{B1}$ and $R_{B2}$.
Terminology

The following are the same:

- Biasing point
- Quiescent point
- Operating point (OP)
- DC point
Alternative view of biasing

Load line
Slope=$-1/R_L$

operating point

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What controls the operating point?

**CONCLUSION:** $V_{BE}$ or $I_B$ controls the OP. $R_L$ also controls the OP.
What happens if $V_{BE}$ dances up and down?

The OP also dances up and down along the load line. $V_{CE}$ also moves up and down.

Typically, when $V_{BE}$ moves a little bit, $V_{CE}$ moves a lot! THIS IS CALLED AMPLIFICATION.
Animation to show amplifier action
Derivation of voltage gain

Question: what is \( \frac{\Delta V_o}{\Delta V_{in}} = \frac{\Delta V_{CE}}{\Delta V_{BE}} \) ?

Clearly, Ohm’s law says that

\[ V_{CE} = V_{CC} - I_C R_L \quad \Rightarrow \quad \Delta V_{CE} = -R_L \Delta I_C \]

Then, what relates \( \Delta I_C \) and \( \Delta V_{BE} \)?

Last lecture: transconductance

\[ g_m = \frac{\Delta I_C}{\Delta V_{BE}} \]

Hence,

\[ \frac{\Delta V_{CE}}{\Delta V_{BE}} = -g_m R_L \]
The one we have just studied is called COMMON-EMITTER amplifier.

Small-signal voltage gain
\[ = -g_m R_L \]

That means we can increase the gain by increasing \( g_m \) and/or \( R_L \).

Output waveform is anti-phase.
How do we inject signal into the amplifier?

\[ V_{CE} = V_{CE} \pm \Delta v_{CE} \]

or

\[ \Delta v_{in} \]

\[ \pm 20\text{mV} \]
Note on symbols

\[ v_{CE} = V_{CE} + \tilde{v}_{CE} \]

- Total signal
- Small signal
- DC point
- \( \Delta a \) or \( \tilde{a} \)
Solution: Add the **same** biasing DC level

But, it is impossible to find a voltage source which is equal to the exact biasing voltage across B-E.

$V_{BE}$ could actually be 0.621234V, which is determined by the network RB1, RB2 and the transistor characteristic!!

How to apply the exact $V_{BE}$?
The wonderful voltage source: capacitor

The capacitor voltage is exactly equal to $V_{BE}$ because DC current must be zero.
Solution — insert coupling capacitor

DC voltage equal to exactly the same biasing $V_{BE}$

This is called a coupling capacitor

$V_{in} \sim \pm 20\text{mV}$
Complete common emitter amplifier

coupling capacitors
(large enough so that they become short-circuit at signal frequencies)
Can we simplify the analysis?

We are mainly interested in the ac signals. The DC bias does not matter!

Can we create a simple circuit just to look at ac signals?
Two basic questions:

1. What is the loading (resistance) seen here?
2. What is the Thévenin or Norton equivalent circuit seen here?
Small-signal model of BJT: objectives

To find: \( r_{in} \), \( R_o \), \( G_m \)

or

\( r_{in} \), \( R_o \), \( A_m \)

Norton form

\( r_{in} \), \( R_o \)

Thévenin form

\( A_m v_{in} \)
Derivation of the small-signal model

Input side:

\[ r_\pi \]

\[ r_{in} = \frac{v_{BE}}{i_B} = \frac{v_{BE}}{i_C / \beta} \]

For small-signal,

\[ r_{in} = \frac{\Delta v_{BE}}{\Delta i_B} = \frac{\Delta v_{BE}}{\Delta i_C / \beta} \]

\[ = \frac{\beta}{(\Delta i_C / \Delta v_{BE})} = \frac{\beta}{g_m} \]

\[ r_\pi = \beta/g_m \]

where \( g_m \) is the BJT’s transconductance.
Derivation of the small-signal model

Output side:

\[ v_{CE} = V_{CC} - I_C R_L \]

For small-signal,

\[ \Delta v_{CE} = -\Delta i_C \times R_L = -g_m \Delta v_{BE} \times R_L \]

where \( g_m \) is the BJT’s transconductance
Derivation of the small-signal model

Output side:

\[ V_{CC} \]
\[ R_L \]
\[ I_C \]
\[ V_{CE} \]

Including BJT’s Early effect

\[ \frac{\Delta V_{CE}}{R_L} + \frac{\Delta V_{CE}}{r_o} = -\Delta I_C \]
\[ \Delta V_{CE} = -\Delta I_C (R_L || r_o) \]
\[ = -g_m \Delta V_{BE} (R_L || r_o) \]

where \( r_o \) is the Early resistor of the BJT.

Recall: \( r_o = \frac{V_A}{I_C} \), where \( V_A \) is typically about 100V.

A very rough approx. is \( r_o = \infty \).
Initial small-signal model for BJT

“MUST” REMEMBER

Small-signal BJT parameters:

$$g_m = \frac{I_C}{(kT / q)} = \frac{I_C}{V_T}$$

$$r_\pi = \frac{\beta}{g_m}$$

$$r_o = \frac{V_A}{I_C}$$

$V_T$ is thermal voltage

$\approx 25mV$

$V_A$ is Early voltage

typically $\sim 100V$
Initial small-signal model for FET

Similar to BJT, but input resistance is $\infty$.

Small-signal FET parameters:

\[ g_m = 2\sqrt{K}\sqrt{I_D} \]

\[ r_o = \frac{1}{\lambda} \]

$\lambda$ is the channel length modulation parameter
$K$ is a semiconductor parameter

All amplifier configurations using BJT can be likewise constructed using FET.
Example: common-emitter amplifier

Assume the coupling caps are large enough to be considered as short-circuit at signal frequency.

$V_{CC}$ is ac 0V.
Complete model for common-emitter amplifier

Complete model:

+ \( V_{\text{in}} \)
  \( R_{B1} \parallel R_{B1} \parallel r_\pi \)
  \( \tilde{V}_{BE} \)
  \( r_m \tilde{V}_{BE} \)
  \( R_L \parallel r_o \)
  \( V_o \)

Simplified model:

+ \( V_{\text{in}} \)
  \( R_{B1} \parallel R_{B1} \parallel r_\pi \)
  \( \tilde{V}_{BE} \)
  \( g_m \tilde{V}_{BE} \)
  \( R_L \parallel r_o \)
  \( V_o \)

Total input resistance
\[
R_{\text{in}} = R_{B1} \parallel R_{B1} \parallel r_\pi
\]

Total output resistance
\[
R_o = R_L \parallel r_o
\]

Voltage gain
\[
\frac{V_o}{V_{\text{in}}} = -g_m (R_L \parallel r_o)
\approx -g_m R_L
\]
Alternative model for common-emitter amplifier

Output in Thévenin form:

Total input resistance

\[ R_{\text{in}} = R_{B1} \parallel R_{B1} \parallel r_{\pi} \]

Total output resistance

\[ R_{o} = R_{L}||r_{o} \]

Voltage gain

\[ \frac{V_{o}}{V_{in}} = -g_{m}(R_{L} \parallel r_{o}) \]

\[ \approx -g_{m}R_{L} \]
More about common-emitter amplifier

Because the output resistance is quite large (equal to $R_L||r_o \approx R_L$), the common-emitter amplifier is a poor voltage driver. That means, it is not a good idea to use such an amplifier for loads which are smaller than $R_L$. This makes it not suitable to deliver current to load.

1kΩ, for example

practically no output!!

10Ω
Bad idea — wrong use of common-emitter amplifier

Transconductance $g_m = I_C / (25 \text{mV}) = 5/25 = 0.2 \text{ A/V}$

Expected gain = $g_m R_L = (0.2)(1k) = 200$ or $46 \text{dB}$

But the output circuit is:

The effective gain drops to

$$200 \times \frac{10}{1000 + 10} = 1.98$$
Proper use of common-emitter amplifier

The load must be much larger than $R_L$.

Now the output circuit is:

The effective gain is

$$200 \times \frac{10^7}{1000 + 10^7} \approx 200$$
How can we use the amplifier in practice?

How to connect the output to load?

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Emitter follower

Biasing conditions:
Base voltage \( \approx 5.6\text{V} \)
Emitter voltage \( \approx 5\text{V} \)
Collector current \( \approx 10\text{mA} \)
\( R_E = 500\Omega \)
\( R_{B1}:R_{B2} \approx 44:56 \)
Say, \( R_{B1} = 440k\Omega \)
\( R_{B2} = 560k\Omega \)

\( V_E = V_B - 0.6 \)
Thus, for small signal,
\[ \Delta V_E = \Delta V_B \]
or
\[ v_o = v_{in} \]

Gain = \( v_o / v_{in} = 1 \)
Small-signal model of emitter follower

\[ +10V \]

\[ R_{B1} \quad R_{B2} \]

\[ + v_{in} \quad - \]

\[ R_E \quad + \quad v_o \quad - \]

\[ R_{B1} || R_{B2} \]

\[ g_m \tilde{v}_{BE} \]

\[ r_\pi \quad r_o \]

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Small-signal model of emitter follower

Input resistance is

\[ r_{in} = \frac{v_{in}}{i_B} = \frac{v_{BE} + v_E}{i_B} \]

\[ = \frac{v_{BE}}{i_B} + \frac{v_E}{i_B} \]

\[ = r_\pi + \frac{v_E}{i_B} \]

\[ = r_\pi + \frac{v_E}{i_E/(1 + \beta)} \]

\[ = r_\pi + (1 + \beta)R_E \]

which is quite large (good)!!
Small-signal model of emitter follower

Output resistance is

\[ r_{out} = \frac{v_m}{i_m} = \frac{-v_{BE}}{i_m} = \frac{-v_{BE}}{i_E - i_B - g_m v_{BE}} \]

\[ = \frac{v_E}{i_E + \frac{v_E}{r_\pi} + g_m v_E} \approx \frac{1}{\frac{1}{R_E} + \frac{1}{r_\pi} + g_m} + \frac{1}{R_E} + g_m \]

\[ = R_E \parallel \frac{1}{g_m} \]

which is quite small (good)!!
Small-signal model of emitter follower

Thevenin form:

\[ V_{in} + r_{\pi}(1+\beta)R_E \]

Large input resistance
Small output resistance
Voltage gain = 1

Draw no current from previous stage
Good for any load
A better “emitter follower”

Input resistance is very LARGE because $R_E = \infty$.

Output resistance is $1/g_m$.

Gain = 1.

This circuit is also called CLASS A output stage. Details to be studied in second year EC2.
Common-emitter amplifier with emitter follower as buffer

- Common-emitter amplifier (high gain)
- Emitter follower (unit gain)

 schematic diagram
FET amplifiers (similar to BJT amplifiers)

- **FET amplifiers** (similar to BJT amplifiers)
- **+10V**
- **$v_{GS}$**
- **$R_{G1}$**
- **$R_L$**
- **$R_{G2}$**
- **$v_{in}$**
- **$v_{o}$**
- **$10\Omega$**
- **common-source amplifier** (high gain $= -g_m R_L$)
- **source follower** (unit gain)

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Further thoughts

Will the biasing resistors affect the gain?

Seems not, because

\[
\text{Gain} = -g_m R_L
\]

which does not depend on \( R_{bias} \).

However, a realistic voltage source has finite internal resistance. This will affect the gain.
Input source with finite resistance

The input has a voltage divider network.

\[ v_{BE} = v_{in} \frac{R_{bias} \parallel r_\pi}{R_{bias} \parallel r_\pi + R_s} \]

Therefore, the gain decreases to

\[ \frac{v_o}{v_{in}} = \frac{R_{bias} \parallel r_\pi}{R_{bias} \parallel r_\pi + R_s} (-g_m R_L) \]

assuming \( r_o \) very large.
Example

By how much does the gain drop?

\[ r_\pi = \beta/g_m = \frac{100}{0.2} = 500 \Omega \]

\[ g_m = \frac{5mA}{25mV} = 0.2A/V \]

Voltage divider attenuation = \( \frac{R_{bias} \parallel r_\pi}{50 + R_{bias} \parallel r_\pi} = \frac{596 \parallel 500}{50 + 596 \parallel 500} = 0.845 \text{ or } -1.463\text{dB} \)

Hence, the gain is reduced to \( 0.845(g_m R_L) = 169 \)
Recall that the best biasing scheme should be $\beta$ independent.

One good scheme is emitter degeneration, i.e., using $R_E$ to fix biasing current directly. Here, since $V_B$ is about 1.6V, as fixed by the base resistor divider, $V_E$ is about 1V.

Therefore, $I_C \approx \frac{V_E}{R_E} = 5\text{mA}$ (no $\beta$ needed!)

Question:
Will this biasing scheme affect the gain?
Common-emitter amplifier with emitter degeneration

Exercise: Find the small-signal gain of this amplifier.

Answer:

\[
\frac{v_o}{v_{in}} = \frac{-g_m R_L}{1 + \left(1 + \frac{1}{\beta}\right) g_m R_E}
\]

\[
\approx \frac{-g_m R_L}{1 + g_m R_E} \approx -\frac{R_L}{R_E}
\]

The gain is MUCH smaller.

We have a good biasing, but a poor gain! Can we improve the gain?
Common-emitter amplifier with emitter by-pass

Add $C_E$ such that the effective emitter resistance becomes zero at signal frequency.

So, this circuit has good biasing, and the gain is still very high!

Gain $= -g_mR_L$

which is unaffected by $R_E$ because effectively $R_E$ is shorted at signal frequency.

$C_E$ is called bypass capacitor.
Summary

Basic BJT model (small-signal ac model):

\[ g_m = \frac{I_C}{(kT/q)} = \frac{I_C}{V_T} \]

\[ r_\pi = \frac{\beta}{g_m} \]

\[ r_o = \frac{V_A}{I_C} \]

\( V_T \) is thermal voltage

\( \approx \ 25mV \)

\( V_A \) is Early voltage

\( \text{typically } \sim 100V \)
Summary

Basic FET model (small-signal ac model):

Similar to the BJT model, but with infinite input resistance.

Therefore, the FET can be used in the same way as amplifiers.
Common-emitter (CE) amplifier
small-signal ac model:

\[
\begin{align*}
&\text{Gain} = -g_m R_L \\
&\text{Input resistance} = R_{\text{bias}} \parallel r_\pi \quad (\text{quite large — desirable}) \\
&\text{Output resistance} = R_L \parallel r_o \approx R_L \quad (\text{large — undesirable})
\end{align*}
\]
Summary

Emitter follower (EF)
small-signal ac model:

Gain = 1

Input resistance = \( R_{\text{bias}} \parallel \left[ r_\pi + (1+\beta)R_E \right] \) (quite large — desirable)

Output resistance = \( R_E \parallel \left( 1/g_m \right) \) (small — desirable)