Two Port Characterizations

Contents
- Input and output resistances
- Two port networks
- Models
Impedances and loading effects

Voltage amplifiers

- $V_s$ to $R_s$
- $+V_{in}$ to $R_{in}$
- $-V_{in}$ to $A_v V_{in}$
- $A_v V_{in}$ to $R_{out}$
- $R_{out}$ to $+V_o$
- $-V_o$ to $R_{LOAD}$

- $R_s$: larger the better (the best is $\infty$)
- $R_{LOAD}$: smaller the better (the best is 0)
Impedances and loading effects

Current amplifiers

$$A_{in} i_{in}$$

$$R_{in}$$

$$R_{out}$$

larger the better (the best is $\infty$)

$$R_{LOAD}$$

$$i_{out}$$

$$i_{in}$$

smaller the better (the best is 0)
Transconductance amplifiers

\[ G_m v_{in} \]

\[ R_{in} \]

\[ R_{out} \]

\[ R_{LOAD} \]

\[ i_{out} \]

larger the better (the best is \( \infty \))
Impedances and loading effects

Transresistance amplifiers

\[ i_{\text{in}} \]

smaller the better (the best is 0)

\[ R_{\text{LOAD}} \]

smaller the better (the best is 0)
Finding impedances

Input impedance

Inject a current to the input, find the voltage. The ratio of the voltage to current gives the input resistance.

\[ \frac{v_x}{i_x} \]

\[ R_{\text{in}} \]

WITH Output
open-circuit if it is supposed to be a voltage output (e.g., voltage amplifiers and transresistance amplifiers)

short-circuit if it is supposed to be a current output (current amplifiers and transconductance amplifiers)
Output impedance

Inject a current at output, find the voltage. The ratio of the voltage to current gives the output resistance.

WITH input short-circuit if it is supposed to be a voltage input (e.g., voltage amplifiers and transconductance amplifiers)

open-circuit if it is supposed to be a current input (current amplifiers and transresistance amplifiers)
Example: CE amplifier with ED

Input resistance is

\[ r_{\text{in}} = \frac{v_{\text{in}}}{i_B} = \frac{v_{\text{BE}} + v_E}{i_B} \]
\[ = \frac{v_{\text{BE}}}{i_B} + \frac{v_E}{i_B} \]
\[ = r_\pi + \frac{v_E}{i_B} \]
\[ = r_\pi + \left(1 + \beta\right)R_E \]
Example: EF amplifier

Output resistance is

\[ r_{out} = \frac{v_m}{i_m} = \frac{-V_{BE}}{i_m} = \frac{i_E - i_B - g_m V_{BE}}{1} \]

\[ i_E + \frac{v_E}{r_\pi} + g_m V_E \]

\[ = \frac{1}{R_E} + \frac{1}{r_\pi} + g_m \]

\[ \approx \frac{1}{R_E + g_m} \]

\[ = R_E \parallel \frac{1}{g_m} \]
Quick rule 1

\[ r_π + (1 + \beta)R_E \]
Example

\[ r_\pi + (1 + \beta)[R_{E1} || (r_\pi + (1 + \beta)R_{E2})] \]
Quick rule 2

\[ \frac{1}{g_m} + \frac{R_B}{1 + \beta} \]

Prof. C.K. Tse: 2-port networks
Example

\[
\begin{pmatrix}
1 \\
\end{pmatrix}
\begin{pmatrix}
R_L & R_B \\
\end{pmatrix}
\begin{pmatrix}
\frac{1}{g_m} + \frac{R_L + R_B}{1 + \beta} \\
1 \\
\end{pmatrix} \parallel R_E
\]
General two port characterizations

\[ v_1 + i_1 = v_2 - i_2 \]
Types of characterizations

Immittance parameters
- z-parameters
- y-parameters

Hybrid parameters
- h-parameters
- g-parameters
z-parameters

\[
\begin{bmatrix}
    v_1 \\
v_2
\end{bmatrix}
= \begin{bmatrix}
    z_{11} & z_{12} \\
    z_{21} & z_{22}
\end{bmatrix}
\begin{bmatrix}
i_1 \\
i_2
\end{bmatrix}
\]

- \( z_{11} = \frac{v_1}{i_1} \) \( i_2 = 0 \) (open-circuit port 2)
- \( z_{12} = \frac{v_1}{i_2} \) \( i_1 = 0 \) (open-circuit port 1)
- \( z_{21} = \frac{v_2}{i_1} \) \( i_2 = 0 \) (open-circuit port 2)
- \( z_{22} = \frac{v_2}{i_2} \) \( i_1 = 0 \) (open-circuit port 2)
y-parameters

\[
\begin{bmatrix}
    i_1 \\
    i_2
\end{bmatrix} =
\begin{bmatrix}
y_{11} & y_{12} \\
y_{21} & y_{22}
\end{bmatrix}
\begin{bmatrix}
v_1 \\
v_2
\end{bmatrix}
\]

\[
y_{11} = \frac{i_1}{v_1} \quad v_2 = 0 \text{ (short-circuit port 2)}
\]

\[
y_{12} = \frac{i_1}{v_2} \quad v_1 = 0 \text{ (short-circuit port 1)}
\]

\[
y_{21} = \frac{i_2}{v_1} \quad v_2 = 0 \text{ (short-circuit port 2)}
\]

\[
y_{22} = \frac{i_2}{v_2} \quad v_1 = 0 \text{ (short-circuit port 2)}
\]
h-parameters

BJT model in some books:

- $h_{11} = r_{\pi}$ \hspace{1cm} \text{input resistance}
- $h_{21} = h_{fe} = \beta$
- $h_{22} = 1/r_o \hspace{1cm} \text{Early resistance}$

\[
\begin{bmatrix}
    v_1 \\
    i_2
\end{bmatrix} =
\begin{bmatrix}
    h_{11} & h_{12} \\
    h_{21} & h_{22}
\end{bmatrix}
\begin{bmatrix}
    i_1 \\
    v_2
\end{bmatrix}
\]

- $h_{11} = \frac{v_1}{i_1} \hspace{1cm} v_2=0 \hspace{1cm} \text{(short-circuit port 2)}$
- $h_{12} = \frac{v_1}{v_2} \hspace{1cm} i_1=0 \hspace{1cm} \text{(open-circuit port 1)}$
- $h_{21} = \frac{i_2}{i_1} \hspace{1cm} v_2=0 \hspace{1cm} \text{(short-circuit port 2)}$
- $h_{22} = \frac{i_2}{v_2} \hspace{1cm} i_1=0 \hspace{1cm} \text{(open-circuit port 1)}$
g-parameters

\[
\begin{bmatrix}
  i_1 \\
  v_2
\end{bmatrix} =
\begin{bmatrix}
  g_{11} & g_{12} \\
  g_{21} & g_{22}
\end{bmatrix}
\begin{bmatrix}
  v_1 \\
  i_2
\end{bmatrix}
\]

\begin{align*}
g_{11} &= \frac{i_1}{v_1} \quad \text{for}\quad i_2 = 0 \quad \text{(open-circuit port 2)} \\
g_{12} &= \frac{i_1}{i_2} \quad \text{for}\quad v_1 = 0 \quad \text{(short-circuit port 1)} \\
g_{21} &= \frac{v_2}{v_1} \quad \text{for}\quad i_2 = 0 \quad \text{(open-circuit port 2)} \\
g_{22} &= \frac{v_2}{i_2} \quad \text{for}\quad v_1 = 0 \quad \text{(short-circuit port 1)}
\end{align*}

\begin{align*}
\text{(open-circuit port 2):} & \quad i_2 = 0 \\
\text{(short-circuit port 1):} & \quad v_1 = 0
\end{align*}
Example

\[ h_{11} = \frac{v_1}{i_1} \bigg|_{v_2=0} = R_1 \parallel R_2 = \frac{R_1 R_2}{R_1 + R_2} \]

\[ h_{12} = \frac{v_1}{v_2} \bigg|_{i_1=0} = \frac{R_2}{R_1 + R_2} \]

\[ h_{21} = \frac{i_2}{i_1} \bigg|_{v_2=0} = \frac{-R_2}{R_1 + R_2} \]

\[ h_{22} = \frac{i_2}{v_2} \bigg|_{i_1=0} = \frac{1}{R_1 + R_2} \]
Connecting two-ports — series-series

Total \([Z]_T = [Z] + [Z']\)
Connecting two-ports — shunt-shunt

\[ \text{Total } [Y]_T = [Y] + [Y'] \]
Connecting two-ports — shunt-series

Total $[G]_T = [G] + [G']$
Connecting two-ports — series-shunt

\[ [H]_T = [H] + [H'] \]
We can develop circuit model for each type of two-port descriptions.

Example: h-parameter

\[
\begin{bmatrix}
  v_1 \\
  i_2
\end{bmatrix}
= 
\begin{bmatrix}
  h_{11} & h_{12} \\
  h_{21} & h_{22}
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  v_2
\end{bmatrix}
\Rightarrow \begin{cases}
  v_1 = h_{11}i_1 + h_{12}v_2 \\
  i_2 = h_{21}i_1 + h_{22}v_2
\end{cases}
\]
Example: BJT model

We can model the BJT as a h-parameter model:

\[ h_{11} = r_{\pi} \quad \text{input resistance} \]
\[ h_{12} \approx 0 \]
\[ h_{21} = h_{fe} = \beta \]
\[ h_{22} = \frac{1}{r_o} \quad \text{Early resistance} \]