EIE209 Basic Electronics

Basic Transistor Amplifiers

Contents
• Biasing
• Amplification principles
• Small-signal model development for BJT
Aim of this chapter

To show how transistors can be used to amplify a signal.
Basic idea

Step 1: Set the transistor at a certain DC level — biasing

Step 2: Inject a small signal to the input and get a bigger output — coupling
Biasing the transistor

To set the transistor to a certain DC level

Suppose we want the following biasing condition:

\[ I_C = 10 \text{ mA} \quad \text{and} \quad V_{CE} = 5 \text{ V} \]

Find \( R_B \) and \( R_L \)

Start with \( V_{BE} \approx 0.7 \text{ V} \).

Then,

\[ I_B = \frac{10 - V_{BE}}{R_B} = \frac{10 - 0.7}{R_B} \]

\[ I_C = \beta I_B = 100 \frac{10 - 0.7}{R_B} = 10 \text{ mA} \]

So, \( R_B = 94k\Omega \)

Also,

\[ V_{CE} = 10 - R_L I_C \]

Hence,

\[ 5 = 10 - 10R_L \]

So, \( R_L = 0.5k\Omega \)
dependent biasing — bad biasing

Now, let’s go to the lab and try using \( R_B = 94\, \Omega \) and \( R_L = 0.5\, \Omega \), and see if we get what we want.

...totally wrong! We don’t get \( I_C = 10\, \text{mA} \) and \( V_{CE} = 5\, \text{V} \)

This is a bad biasing circuit!

because it relies on the accuracy of \( b \), but \( b \) can be ±50% different from what is given in the databook.
A slightly better biasing method

Again, our objective is to find the resistors such that $I_C = 10\text{mA}$ and $V_{CE} = 5\text{V}$.

First, if $I_B$ is **small**, we can approximately write

$$0.6 = 10 \frac{R_{B2}}{R_{B1} + R_{B2}}$$

Suppose we get $I_C = 10\text{mA}$. Then $R_L = 0.5\text{k\Omega}$. We can start with $R_{B1} = 940\Omega$ and $R_{B2} = 60\Omega$. Such resistors will make sure $I_B$ is **much smaller** than the current flowing down $R_{B1}$ and $R_{B2}$, which is consistent with the assumption.

What we need in practice is to fine tune $R_{B1}$ or $R_{B2}$ such that $V_{CE}$ is exactly 5V.
A much better biasing method — emitter degeneration

Again, our objective is to find the resistors such that $I_C = 10\text{mA}$ and $V_{CE} = 5\text{V}$.

Set $V_E = 2\text{V}$, say. Then, $R_E = 2\text{V}/10\text{mA} = 0.2\text{k}\Omega$.

Surely, $R_L = 0.5\text{k}\Omega$ in order to get $V_{CE} = 5\text{V}$.

Finally, we have $V_B = V_E + 0.6$. Therefore, if $I_B$ is small compared to $I_{RB1}$ and $I_{RB2}$, we have

$$\frac{R_{B1}}{R_{B2}} = \frac{74}{26}$$

Hence, $R_{B1} = 740\Omega$ and $R_{B1} = 260\Omega$.

**NOTE:** $\square$ is never used in calculation!!
Stable (good) biasing

Summary of biasing with emitter degeneration:

Choose $V_E$, $I_C$ and $V_{CE}$.

Use $V_{BE} \approx 0.6$ to get $V_B$.

Then use

$$\frac{R_{B1}}{R_{B2}} = \frac{10}{V_B}$$

to choose $R_{B1}$ and $R_{B2}$ such that $I_B$ is much smaller the current flowing in $R_{B1}$ and $R_{B2}$.
Terminology

The following are the same:

Biasing point

Quiescent point

Operating point (OP)

DC point
Alternative view of biasing

Load line
Slope=$-1/R_L$
operating point
What controls the operating point?

CONCLUSION: $V_{BE}$ or $I_B$ controls the OP

$R_L$ also controls the OP
What happens if $V_{BE}$ dances up and down?

The OP also dances up and down along the load line. $V_{CE}$ also moves up and down.

Typically, when $V_{BE}$ moves a little bit, $V_{CE}$ moves a lot! **THIS IS CALLED AMPLIFICATION.**
Animation to show amplifier action
Derivation of voltage gain

Question: what is \( \frac{\Delta V_o}{\Delta V_{in}} = \frac{\Delta V_{CE}}{\Delta V_{BE}} \) ?

Clearly, Ohm’s law says that

\[ V_{CE} = V_{CC} - I_C R_L \quad \Rightarrow \quad \Delta V_{CE} = R_L \cdot \Delta I_C \]

Then, what relates \( \Delta I_C \) and \( \Delta V_{BE} \)?

Last lecture: transconductance

\[ g_m = \frac{\Delta I_C}{\Delta V_{BE}} \]

Hence, \( \frac{\Delta V_{CE}}{\Delta V_{BE}} = g_m R_L \)
Common-emitter amplifier

The one we have just studied is called COMMON-EMITTER amplifier.

SUMMARY:

Small-signal voltage gain
\[ = -g_m R_L \]

That means we can increase the gain by increasing \( g_m \) and/or \( R_L \).

Output waveform is anti-phase.
How do we inject signal into the amplifier?

\[ V_{in} \pm 20\text{mV} \]

\[ v_{in} \sim \]

\[ V_{in} \]

\[ v_{in} \]

\[ I_C \]

\[ v_{CE} = V_{CE} + \sim v_{CE} \]

\[ R_{B1} \]

\[ R_{B2} \]

\[ R_L \]

\[ V_{CC} \]

\[ V_{BE} \]

\[ V_{CE} \]
Note on symbols

\[ v_{CE} = V_{CE} + \tilde{v}_{CE} \]

- **Total signal**
- **DC point**
- **Small signal**

\[ a \text{ or } \tilde{a} \]
Solution: Add the same biasing DC level

Exactly the same biasing $V_{BE}$

$V_{in}$ ±20mV

But, it is impossible to find a voltage source which is equal to the exact biasing voltage across B-E.

$V_{BE}$ could actually be 0.621234V, which is determined by the network $R_{B1}$, $R_{B2}$ and the transistor characteristic!!

How to apply the exact $V_{BE}$?
The wonderful voltage source: capacitor

The capacitor voltage is exactly equal to $V_{BE}$ because DC current must be zero.
Solution — insert coupling capacitor

This is called a coupling capacitor

DC voltage equal to exactly the same biasing $V_{BE}$

$\sim V_{in}$
±20mV

±20mV
Complete common emitter amplifier

coupling capacitors (large enough so that they become short-circuit at signal frequencies)
Can we simplify the analysis?

We are mainly interested in the ac signals. The DC bias does not matter!

Can we create a simple circuit just to look at ac signals?
Small-signal model

Two basic questions:

1. What is the loading (resistance) seen here?

2. What is the Thévenin or Norton equivalent circuit seen here?
Small-signal model of BJT: objectives

To find: $r_{in}$, $R_o$, $G_m$ or $r_{in}$, $R_o$, $A_m$

Norton form

Thévenin form
**Derivation of the small-signal model**

**Input side:**

\[ r_{in} = \frac{v_{BE}}{i_B} = \frac{v_{BE}}{i_C} \]

For small-signal,

\[ r_{in} = \frac{\Delta v_{BE}}{\Delta i_B} = \frac{\Delta v_{BE}}{\Delta i_C} \]

\[ = \frac{\Delta}{(\Delta i_C / \Delta v_{BE})} = \frac{\Delta}{g_m} \]

\[ r_{\square} = \frac{1}{g_m} \]

where \( g_m \) is the BJT’s transconductance.
Derivation of the small-signal model

\[ V_{CE} = V_{CC} - I_C R_L \]

For small-signal,
\[ \Delta V_{CE} = \Delta I_C \Delta R_L \\
= g_m \Delta V_{BE} \Delta R_L \]

where \( g_m \) is the BJT’s transconductance
Derivation of the small-signal model

Including BJT’s Early effect

\[
\frac{\Delta V_{CE}}{R_L} + \frac{\Delta V_{CE}}{r_o} = \Delta i_C \\
\Delta V_{CE} = \Delta i_C (R_L \parallel r_o) \\
= g_m \Delta V_{BE} (R_L \parallel r_o)
\]

where \( r_o \) is the Early resistor of the BJT.

Recall: \( r_o = V_A / I_C \), where \( V_A \) is typically about 100V.

A very rough approx. is \( r_o = \infty \).
Initial small-signal model for BJT

“MUST” REMEMBER

BJT model

Small-signal BJT parameters:

\[ g_m = \frac{I_C}{(kT/q)} = \frac{I_C}{V_T} \]

\[ r_\square = \frac{\square}{g_m} \]

\[ r_o = \frac{V_A}{I_C} \]

\( V_T \) is thermal voltage

\( \square \) 25mV

\( V_A \) is Early voltage

typically \( \sim 100V \)
Initial small-signal model for FET

Similar to BJT, but input resistance is $\infty$.

Small-signal FET parameters:

\[ g_m = 2\sqrt{K} \sqrt{I_D} \]
\[ r_o = \frac{1}{\square} \]

\( \square \) is the channel length modulation parameter

\( K \) is a semiconductor parameter

All amplifier configurations using BJT can be likewise constructed using FET.
Example: common-emitter amplifier

Assume the coupling caps are large enough to be considered as short-circuit at signal frequency.

\[ V_{CC} \]

\[ R_{B1} \]

\[ R_L \]

\[ V_{BE} \]

\[ + \]

\[ - \]

\[ + \]

\[ - \]

\[ V_{in} \]

\[ r_o \]

\[ g_m \tilde{v}_{BE} \]

\[ R_{B1} || R_{B1} \]

\[ R_{B2} \]

\[ V_{CC} \text{ is ac 0V.} \]

Prof. C.K. Tse: Amplifier Configurations
Complete model for common-emitter amplifier

Complete model:

\[ + \quad V_{\text{in}} \quad R_{B1} \parallel R_{B1} \quad + \quad \tilde{V}_{BE} \quad r_{\parallel} \quad g_{m} \tilde{V}_{BE} \quad r_{o} \quad R_{L} \quad V_{o} \quad + \]

Simplified model:

\[ + \quad V_{\text{in}} \quad R_{B1} \parallel R_{B1} \parallel r_{\parallel} \quad \tilde{V}_{BE} \quad + \quad g_{m} \tilde{V}_{BE} \quad R_{L} \parallel r_{o} \quad V_{o} \quad + \]

Total input resistance

\[ R_{in} = R_{B1} \parallel R_{B1} \parallel r_{\parallel} \]

Total output resistance

\[ R_{o} = R_{L} \parallel r_{o} \]

Voltage gain

\[ \frac{V_{o}}{V_{\text{in}}} = g_{m} (R_{L} \parallel r_{o}) \]

\[ = g_{m} R_{L} \]
Alternative model for common-emitter amplifier

Output in Thévenin form:

Total input resistance

\[ R_{in} = R_{B1} \parallel R_{B1} \parallel r_p \]

Total output resistance

\[ R_o = R_L \parallel r_o \]

Voltage gain

\[ \frac{V_o}{V_{in}} = -g_m (R_L \parallel r_o) \]

\[ \parallel g_m R_L \]
More about common-emitter amplifier

Because the output resistance is quite large (equal to $R_L \parallel r_o \approx R_L$), the common-emitter amplifier is a POOR voltage driver. That means, it is not a good idea to use such an amplifier for loads which are smaller than $R_L$. *This makes it not suitable to deliver current to load.*

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Bad idea — wrong use of common-emitter amplifier

Transconductance \( g_m = \frac{I_C}{25 \text{mV}} = \frac{5}{25} = 0.2 \text{ A/V} \)

Expected gain = \( g_m R_L = (0.2)(1 \text{k}) = 200 \text{ or } 46 \text{dB} \)

But the output circuit is:

The effective gain drops to

\[
\frac{200 \cdot \frac{10}{1000+10}} = 1.98
\]
Proper use of common-emitter amplifier

The load must be much larger than $R_L$. The effective gain is

\[
\frac{200V_{in}}{10M\Omega} \cdot \frac{10^7}{1000+10^7} \approx 200
\]
How can we use the amplifier in practice?

How to connect the output to load?

Prof. C.K. Tse: Amplifier Configurations
Emitter follower

Biasing conditions:
Base voltage $\approx 5.6V$
Emitter voltage $\approx 5V$
Collector current $\approx 10mA$
$R_E = 500\,\Omega$
$R_{B1}:R_{B2} \approx 44:56$
Say, $R_{B1} = 440k\Omega$
$R_{B2} = 560k\Omega$

Thus, for small signal,

$V_E = V_B - 0.6$

Gain $= \frac{v_o}{v_{in}} = 1$
Small-signal model of emitter follower

$$+10V$$

$$R_{B1}$$

$$R_{B2}$$

$$R_E$$

$$R_{B1} \parallel R_{B2}$$

$$R_E$$

$$g_m \tilde{v}_{BE}$$

$$r_D$$

$$r_o$$

$$v_{in}$$

$$v_o$$

$$v_o$$

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Small-signal model of emitter follower

Input resistance is

\[
 r_{in} = \frac{v_{in}}{i_B} = \frac{v_{BE} + v_E}{i_B} \\
 = r_D + \frac{v_E}{i_B} \\
 = r_D + \frac{v_E}{i_E/(1 + \beta)} \\
 = r_D + (1 + \beta)R_E
\]

which is quite large (good)!!
Small-signal model of emitter follower

Output resistance is

\[ r_{out} = \frac{v_m}{i_m} = \frac{v_{BE}}{i_m} = \frac{i_E - i_B - g_m v_{BE}}{i_E + \frac{v_E}{r_o} + g_m v_E} \]

\[ = \frac{1}{r_o} + \frac{1}{R_E} + g_m \]

\[ = R_E \parallel \frac{1}{g_m} \]

which is quite small (good)!!
Small-signal model of emitter follower

Thevenin form:

\[ v_{in} \]

\[ r_o + (1 + \frac{1}{g_m}) R_E \]

\[ R_E \parallel (1/g_m) \]

\[ v_{in} - v_{o} \]

Large input resistance
Small output resistance
Voltage gain = 1

Draw no current from previous stage
Good for any load
A better “emitter follower”

Input resistance is very LARGE because $R_E = \infty$.

Output resistance is $1/g_m$.

Gain = 1.

This circuit is also called **CLASS A output stage**. Details to be studied in second year EC2.
Common-emitter amplifier with emitter follower as buffer

common-emitter amplifier (high gain)

emitter follower (unit gain)

$V_{BE}$

$R_{B1}$

$R_L$

$V_{in}$

$V_o$

$1/g_m$

speaker 10Ω

$10V$
FET amplifiers (similar to BJT amplifiers)

FET amplifiers (similar to BJT amplifiers)
Further thoughts

Will the biasing resistors affect the gain?

Seems not, because

\[ \text{Gain} = -g_m R_L \]

which does not depend on \( R_{\text{bias}} \).

However, a realistic voltage source has finite internal resistance. This will affect the gain.
Input source with finite resistance

The input has a voltage divider network.

\[ v_{BE} = v_{in} \frac{R_{bias} \parallel r_o}{R_{bias} \parallel r_o + R_s} \]

Therefore, the gain decreases to

\[ \frac{v_o}{v_{in}} = \frac{R_{bias} \parallel r_o}{R_{bias} \parallel r_o + R_s} (g_m R_L) \]

assuming \( r_o \) very large.
Example

By how much does the gain drop?

\[ g_m = \frac{5 \text{mA}}{25 \text{mV}} = 0.2 \text{A/V} \]

\[ r_D = \frac{g_m}{g_m} = \frac{100}{0.2} = 500 \Omega \]

Voltage divider attenuation = \[ \frac{R_{\text{bias}} || r_D}{50 + R_{\text{bias}} || r_D} = \frac{94k \Omega || 600 \Omega}{50 + 94k \Omega || 1k \Omega} = 0.845 \text{ or } -1.463 \text{dB} \]

Hence, the gain is reduced to \[ 0.845(g_m R_L) = 169 \]
Further thoughts

Recall that the best biasing scheme should be $I_B$ independent.

One good scheme is emitter degeneration, i.e., using $R_E$ to fix biasing current directly. Here, since $V_B$ is about 1.6V, as fixed by the base resistor divider, $V_E$ is about 1V.

Therefore, $I_C \approx V_E/R_E = 5mA$ (no $I_B$ needed!)

Question:
Will this biasing scheme affect the gain?
Common-emitter amplifier with emitter degeneration

Exercise: Find the small-signal gain of this amplifier.

Answer:

\[
\frac{v_o}{v_{in}} = \frac{g_m R_L}{1 + \frac{1}{g_m R_E} + \frac{g_m R_L}{1 + g_m R_E}}
\]

The gain is MUCH smaller.

We have a good biasing, but a poor gain! Can we improve the gain?
Add $C_E$ such that the effective emitter resistance becomes zero at signal frequency.

So, this circuit has good biasing, and the gain is still very high!

Gain = $-g_m R_L$

which is unaffected by $R_E$ because effectively $R_E$ is shorted at signal frequency.

$C_E$ is called bypass capacitor.
Summary

Basic BJT model
(small-signal ac model):

\[ g_m = \frac{I_C}{(kT/q)} = \frac{I_C}{V_T} \]

\[ r\| = \frac{1}{g_m} \]

\[ r_o = \frac{V_A}{I_C} \]

\( V_T \) is thermal voltage
- \( \approx 25 \text{mV} \)
\( V_A \) is Early voltage
- typically \( \sim 100 \text{V} \)
Summary

Basic FET model (small-signal ac model):

Similar to the BJT model, but with infinite input resistance.

Therefore, the FET can be used in the same way as amplifiers.
Summary

Common-emitter (CE) amplifier
small-signal ac model:

Gain = $-g_m R_L$

Input resistance = $R_{bias} \parallel r_o$ (quite large — desirable)

Output resistance = $R_L \parallel r_o \approx R_L$ (large — undesirable)
Summary

Emitter follower (EF) small-signal ac model:

Gain = 1

Input resistance = \( R_{\text{bias}} \parallel [r_{\Box} + (1 + [\Box])R_E] \) (quite large — desirable)

Output resistance = \( R_E \parallel (1/g_m) \) (small — desirable)